

Syllabi of Papers offered at M. Sc. Part I (Mathematics) w. e. f. 2009.

**Paper 401 Real Analysis**

L	P	T	Total
4	0	0	4

Lebesgue Measure and Lebesgue Integral:

Introduction, outer measure, measurable sets and Lebesgue measure, a non-measurable set, measurable functions, Littlewood's three principles, the Riemann integral, the Lebesgue integral of a bounded function over a set of finite measure, the integral of a non-negative function, the general Lebesgue integral, convergence in measure.

Differentiation and Integration:

Differentiation of monotone functions, functions of bounded variation, Differentiation of an integral, absolute continuity, convex functions.

The Classical Banach Spaces:

The  $L^p$  spaces, convergence and completeness, approximations in  $L^p$ , bounded linear functions on  $L^p$  spaces.

Measure and Integration:

Measure spaces, measurable functions, signed measures, Hahn decomposition theorem, Jordan decomposition theorem, Lebesgue decomposition theorem.

Measure and Outer Measure:

Outer measure and measurability, the extension theorem, product measure, inner measure, Carathéodory outer measure, Hausdorff measure.

References:

- [1] H. L. Royden, Real Analysis, Macmillan Pub. Co. Inc. 4th Edition, New York, 1993.
- [2] Walter Rudin, Principles of Mathematical Analysis (3rd Edition) Mc Graw Hill, Kogakusha, 1976, International Student Edition.
- [3] T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985
- [4] G. de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.
- [5] I. P. Natanson, Theory of Functions of a Real Variable, Vol. I, Frederick Ungar Publishing Co. 1961

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**Paper 402 Functional Analysis**

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Normed Spaces and Banach Spaces:

Vector spaces, Brief introduction to convergence and completeness in  $\mathbb{R}^n$ . Introduction to basic results of Linear Algebra, definition of norm and Banach space with illustrations, Properties of normed spaces, Theorem of completion on normed spaces (without proof), Finite dimensional normed spaces and subspaces, Compactness and finite dimension, Riesz lemma, Linear operators and their illustrations, Bounded continuous linear operators and their illustrations, Linear functionals and illustrations, Algebraic dual space and reflexivity, Functionals on finite dimensional spaces, Normed spaces of operators and dual space.

Inner Product Spaces-Hilbert Spaces:

Inner product space - definition and illustrations, Hilbert space, Properties of inner product spaces, Orthogonal complements and direct sums, Orthogonal sets, sequences and related series, Total orthonormal sets and sequences, Functionals on Hilbert spaces, Riesz's theorem, Sesquilinear form and Riesz representation theorem, Hilbert adjoint operator and its properties, Self adjoint, unitary and normal operators.

Fundamental theorem for Normed and Banach Spaces:

Zorn's Lemma, Hahn-Banach theorem, Generalised Hahn-Banach theorem, Hahn-Banach theorem for Normed spaces, Application to bounded linear functionals on  $C[a, b]$ , Riesz's theorem for functionals on  $C[a, b]$ , Adjoint operator and its norm, Relation between adjoint and Hilbert adjoint operators, Reflexive spaces, Baire's Category theorem and Uniform Boundedness theorem with applications, Strong and Weak convergence, Convergence of sequences of operators and functionals, Open Mapping theorem, Closed linear operators and Closed Graph theorem.

Contraction Mapping Principle:

Banach fixed-point theorem, Application of Banach fixed-point theorem to system of linear equations, differential equations and integral equations.

References:

- [1] Introduction to Functional Analysis with Applications by Erwin Kreyszig, John Wiley & Sons, 1978.
- [2] Functional Analysis by B. V. Limaye, 2nd ed, New age Int. Pvt. Ltd.
- [3] Introduction to Topology and 'Modern Analysis by G. F. Simmons, McGraw Hill Book Co.
- [4] Functional Analysis by Koffman and Patrie.
- [5] Functional Analysis by A. H. Siddiqui, Prentice Hall of India.

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**Paper 403 Topology**

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Metric Spaces:

The definition and, some examples, open sets, closed sets, convergence, completeness, and Baire's theorem, continuous mapping, spaces of continuous functions, Euclidean and unitary spaces.

Topological Spaces:

The definition and some examples, elementary concepts, open bases and sub-bases, weak topologies, the function algebras of  $C(X, \mathbb{R})$ , and  $C(X, \mathbb{C})$ .

Compactness:

Compact spaces, product spaces, Tychonoff's theorem and locally compact spaces, compactness of metric spaces, Ascoli's theorem.

Separation:

Separation axioms,  $T_0$ ,  $T_1$ ,  $T_2$ ,  $T_{3/2}$ ,  $T_4$ ,  $T_1$ -spaces and Hausdorff spaces, completely regular spaces and normal spaces, Urysohn's lemma and the Tietze extension theorem, the Urysohn's imbedding theorem, the Stone-Cech compactification.

Connectedness:

Connected spaces, the components of a space, totally disconnected spaces, locally connected spaces.

Approximation:

The Weierstrass approximation theorem, the Stone-Weierstrass theorem, locally compact Hausdorff spaces, the extended Stone-Weierstrass theorems.

References:

- [1] George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Co., 1963.
- [2] James R. Munkres, Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
- [3] J. Dugundji, Topology, Allyn and Bacon, 1966 (Reprinted in India by Prentice Hall of India Pvt. Ltd.).
- [4] K. D. Joshi, Introduction to General Topology, Wiley Eastern td., 1983.
- [5] J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1961.

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**Paper 404 Complex Analysis**

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Complex Numbers and Functions:

Introduction, polar form, complex valued functions, limit, complex differentiability, Cauchy-Riemann equations.

Power Series:

Formal power series, convergent power series and relation between them, analytic functions, differentiation of power series.

Cauchy Theorems and Winding Numbers:

Holomorphic functions on connected sets, integrals over paths, local primitive for a Holomorphic function, the Homotopy form of Cauchy's theorem, global primitives, definition of the logarithm, the local Cauchy formula, Winding numbers, the global Cauchy theorem.

Applications of Cauchy Integral Formula & Calculus of Residues:

Uniform limits of analytic functions, Laurent series, isolated singularities, removable singularities, poles and essential singularities, the Cauchy residue theorem, residues of differentials, evaluation of definite integrals, Fourier transforms, trigonometric integrals, Mellin transform.

Harmonic Functions:

Definition, examples, basic properties, the Poisson formula and construction of harmonic functions.

Entire and Meromorphic and elliptic functions:

Jensen's formula and inequality, definition of entire functions, infinite products, Weierstrass products, functions of finite order, meromorphic functions, Mittag-Leffler theorem, the Liouville theorems, the Weierstrass function, the gamma functions.

References:

- [1] S. Lang, Complex Analysis, Addison Wesley, 1997.
- [2] S. Ponnuswamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.
- [3] H. A Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
- [4] J. B. Conway, Functions of one Complex variable, Springer-Verlag, International Student Edition, Narosa Publishing House, 1980.
- [5] Liang-Shin Hahn & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.

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**Paper 405 Ordinary differential equations**

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Existence Uniqueness and Continuation of Solutions:

Introduction, Notation and Definitions, Existence and Uniqueness of Solutions of Scalar Differential Equations, Existence Theorems for System of Differential Equations, Differential and Integral Inequalities.

Linear Systems:

Introduction, Properties of Linear Homogeneous Systems, Inhomogeneous Linear Systems, Behaviour of Solutions of nth Order Linear Homogeneous Equations.

Stability of Linear and Weakly Nonlinear Systems:

Introduction, Continuous Dependence and Stability Properties of Solutions, Linear Systems.

Second Order Differential Equations:

Introduction, Preliminary Results, Boundedness of Solutions, Application to Some Classical Equations.

References:

- [1] Shair Ahmad and M Rama Mohana Rao, Theory of Ordinary Differential Equations, Affiliated East-West Press Pvt. Ltd., New Delhi, 1999.
- [2] P. Hartman, Ordinary Differential Equations, John Wiley, 1964.
- [3] W. T. Reid, Ordinary Differential Equations, John Wiley, New York, 1971.
- [4] E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill, NY, 1955.